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# Quark-Loop Amplitudes for $W^\pm H^\mp$ Associated Hadroproduction

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## Abstract

In this addendum to our paper entitled  *$W^\pm H^\mp$  Associated Production at the Large Hadron Collider* [Phys. Rev. D **59**, 015009 (1999)], we list analytic results for the helicity amplitudes of the partonic subprocess  $gg \rightarrow W^- H^+$  induced by virtual quarks.

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In a recent paper [1], we studied the hadroproduction of a charged Higgs boson in association with a  $W$  boson at the CERN Large Hadron Collider (LHC) in the context of the two-Higgs-doublet model of type II, which serves as the Higgs sector for the minimal supersymmetric extension of the standard model (SM). This reaction dominantly proceeds via the partonic subprocesses  $b\bar{b} \rightarrow W^\pm H^\mp$  at the tree level (see Fig. 1 in Ref. [1]) and  $gg \rightarrow W^\pm H^\mp$ , which is mediated by triangle- and box-type diagrams involving virtual top and bottom quarks (see Fig. 2 in Ref. [1]). In Ref. [1], we presented analytic expressions for the cross section of  $b\bar{b} \rightarrow W^\pm H^\mp$  and the transition-matrix element of  $gg \rightarrow W^\pm H^\mp$  arising from the quark triangles. However, we refrained from listing our formulas for the quark box contributions because we found that were somewhat lengthy. In the meantime, a signal-versus-background analysis of  $W^\pm H^\mp$  associated production at the LHC was carried out by Moretti and Odagiri [2], who generated the signal cross section by using the formulas published in Ref. [1], thus omitting the quark box contributions. This motivated us to further compactify our expressions for the latter by introducing helicity amplitudes. The purpose of this brief report is to provide these results, which may be useful for other authors as well.

Calling the four-momenta of the two gluons and the  $W$  boson  $p_a$ ,  $p_b$ , and  $p_W$ , respectively, we define the partonic Mandelstam variables as  $s = (p_a + p_b)^2$ ,  $t = (p_a - p_W)^2$ , and  $u = (p_b - p_W)^2$ . Furthermore, we introduce the following short-hand notations:  $w = m_W^2$ ,  $h = m_H^2$ ,  $d = t - u$ ,  $t_1 = t - w$ ,  $t_2 = t - h$ ,  $u_1 = u - w$ ,  $u_2 = u - h$ ,  $N = tu - wh$ ,  $\lambda = s^2 + w^2 + h^2 - 2(sw + wh + hs)$ , and  $q = m_t^2 - m_b^2$ . We label the helicity states of the two gluons and the  $W$  boson in the partonic center-of-mass frame by  $\lambda_a = -1/2, 1/2$ ,  $\lambda_b = -1/2, 1/2$ , and  $\lambda_W = -1, 0, 1$ . For reference, we first list the helicity amplitudes for the quark triangle contributions,  $\mathcal{M}_{\lambda_a \lambda_b \lambda_W}^\Delta$ . They may be extracted from Eq. (5) of Ref. [1] and read

$$\mathcal{M}_{\lambda_a \lambda_b 0}^\Delta = \frac{s\sqrt{\lambda}}{m_W} [(1 + \lambda_a \lambda_b) \Sigma(s) - (\lambda_a + \lambda_b) \Pi(s)], \quad (1)$$

where  $\Sigma$  and  $\Pi$  are the vector and axial-vector form factors given in Eq. (6) of Ref. [1]. In this case, the  $W$  boson can only be longitudinally polarized because it couples to two Higgs bosons, so that  $\mathcal{M}_{\lambda_a \lambda_b \lambda_W}^\Delta = 0$  for  $\lambda_W = \pm 1$ . As for the quark box contributions, all twelve helicity amplitudes,  $\mathcal{M}_{\lambda_a \lambda_b \lambda_W}^\square$ , contribute. Due to Bose and weak-isospin symmetry, they are related by

$$\begin{aligned} \mathcal{M}_{\lambda_a \lambda_b 0}^\square(t, u, m_b^2, m_t^2, \tan \beta) &= \mathcal{M}_{\lambda_b \lambda_a 0}^\square(u, t, m_b^2, m_t^2, \tan \beta), \\ \mathcal{M}_{\lambda_a \lambda_b \lambda_W}^\square(t, u, m_b^2, m_t^2, \tan \beta) &= -\mathcal{M}_{\lambda_b \lambda_a \lambda_W}^\square(u, t, m_b^2, m_t^2, \tan \beta), \\ \mathcal{M}_{\lambda_a \lambda_b 0}^\square(t, u, m_b^2, m_t^2, \tan \beta) &= -\mathcal{M}_{-\lambda_a - \lambda_b 0}^\square(t, u, m_t^2, m_b^2, \cot \beta), \\ \mathcal{M}_{\lambda_a \lambda_b \lambda_W}^\square(t, u, m_b^2, m_t^2, \tan \beta) &= \mathcal{M}_{-\lambda_b - \lambda_a - \lambda_W}^\square(u, t, m_t^2, m_b^2, \cot \beta). \end{aligned} \quad (2)$$

Keeping  $\lambda_W = \pm 1$  generic, we thus only need to specify four expressions. These read:

$$\mathcal{M}_{++0}^\square = \frac{2}{m_W s \sqrt{\lambda}} \left[ (m_b^2 \tan \beta + m_t^2 \cot \beta) F_{++}^0 + m_t^2 \cot \beta G_{++}^0 + (t \leftrightarrow u) \right],$$

$$\begin{aligned}
\mathcal{M}_{+-0}^{\square} &= \frac{1}{m_W N \sqrt{\lambda}} \left[ \left( m_b^2 \tan \beta + m_t^2 \cot \beta \right) F_{+-}^0 + m_t^2 \cot \beta G_{+-}^0 \right. \\
&\quad \left. - \left( t \leftrightarrow u, m_b^2 \leftrightarrow m_t^2, \tan \beta \leftrightarrow \cot \beta \right) \right], \\
\mathcal{M}_{++\lambda_W}^{\square} &= \sqrt{\frac{2}{sN}} \left[ \frac{m_b^2 \tan \beta + m_t^2 \cot \beta}{s} \left( \frac{F_{++}^1}{\sqrt{\lambda}} + \lambda_W F_{++}^2 \right) + m_t^2 \cot \beta \left( \frac{G_{++}^1}{\sqrt{\lambda}} + \lambda_W G_{++}^2 \right) \right. \\
&\quad \left. - (t \leftrightarrow u) \right], \\
\mathcal{M}_{+-\lambda_W}^{\square} &= \frac{1}{\sqrt{2sN}} \left[ \frac{m_b^2 \tan \beta + m_t^2 \cot \beta}{N} \left( \frac{F_{+-}^1}{\sqrt{\lambda}} + \lambda_W F_{+-}^2 \right) + m_t^2 \cot \beta \left( \frac{G_{+-}^1}{\sqrt{\lambda}} + \lambda_W G_{+-}^2 \right) \right. \\
&\quad \left. + \left( t \leftrightarrow u, m_b^2 \leftrightarrow m_t^2, \tan \beta \leftrightarrow \cot \beta, \lambda_W \rightarrow -\lambda_W \right) \right], \tag{3}
\end{aligned}$$

where  $F_{+\pm}^i$  and  $G_{+\pm}^i$ , with  $i = 0, 1, 2$ , are complex functions of  $t$ ,  $u$ ,  $m_b^2$ , and  $m_t^2$ . The normalization of  $\mathcal{M}_{\lambda_a \lambda_b \lambda_W}^{\triangle}$  and  $\mathcal{M}_{\lambda_a \lambda_b \lambda_W}^{\square}$  is such that the differential cross section of  $gg \rightarrow W^- H^+$  is given by

$$\frac{d\sigma}{dt}(gg \rightarrow W^- H^+) = \frac{\alpha_s^2(\mu) G_F^2 m_W^2}{256(4\pi)^3 s^2} \sum_{\lambda_a, \lambda_b, \lambda_W} \left| \mathcal{M}_{\lambda_a \lambda_b \lambda_W}^{\triangle} + \mathcal{M}_{\lambda_a \lambda_b \lambda_W}^{\square} \right|^2. \tag{4}$$

We now express the form factors  $F_{+\pm}^i$  and  $G_{+\pm}^i$  in terms of the standard scalar two-, three-, and four-point functions,

$$\begin{aligned}
B_0(p_1^2, m_0^2, m_1^2) &= \int \frac{d^D q}{i\pi^2} \frac{1}{(q^2 - m_0^2 + i\epsilon) [(q + p_1)^2 - m_1^2 + i\epsilon]}, \\
C_0(p_1^2, (p_2 - p_1)^2, p_2^2, m_0^2, m_1^2, m_2^2) &= \int \frac{d^D q}{i\pi^2} \frac{1}{(q^2 - m_0^2 + i\epsilon) [(q + p_1)^2 - m_1^2 + i\epsilon] [(q + p_2)^2 - m_2^2 + i\epsilon]}, \\
D_0(p_1^2, (p_2 - p_1)^2, (p_3 - p_2)^2, p_3^2, p_2^2, (p_3 - p_1)^2, m_0^2, m_1^2, m_2^2, m_3^2) &= \int \frac{d^D q}{i\pi^2} \frac{1}{(q^2 - m_0^2 + i\epsilon) [(q + p_1)^2 - m_1^2 + i\epsilon] [(q + p_2)^2 - m_2^2 + i\epsilon] [(q + p_3)^2 - m_3^2 + i\epsilon]}, \tag{5}
\end{aligned}$$

where  $D$  is the space-time dimensionality. The  $B_0$  function is ultraviolet (UV) divergent in the physical limit  $D \rightarrow 4$ , while the  $C_0$  and  $D_0$  functions are UV finite in this limit. We evaluate the  $B_0$ ,  $C_0$ , and  $D_0$  functions numerically with the aid of the program package FF [3]. To simplify notation, we introduce the abbreviations  $C_{ijk}^{ab}(c) = C_0(a, b, c, m_i^2, m_j^2, m_k^2)$  and  $D_{ijkl}^{abcd}(e, f) = D_0(a, b, c, d, e, f, m_i^2, m_j^2, m_k^2, m_l^2)$ . We find

$$F_{++}^0 = -2s(t_1 + u_1) \left[ m_b^2 C_{bbb}^{00}(s) - m_t^2 C_{ttt}^{00}(s) \right] + f_1(t, u, q) \left[ t_2 C_{btt}^{h0}(t) + t_1 C_{tbb}^{w0}(t) \right]$$

$$\begin{aligned}
& + f_1(u, t, q) \left[ t_2 C_{tbb}^{h0}(t) + t_1 C_{btt}^{w0}(t) \right] - f_1(t, u, q) \left[ N + s(m_b^2 + m_t^2) \right] D_{bttb}^{h0w0}(t, u) \\
& - 2sm_b^2 [2wt_2 + f_1(t, u, q)] D_{btbb}^{hw00}(s, t) + 2sm_t^2 [2wu_2 - f_1(t, u, q)] D_{tbtt}^{hw00}(s, t), \\
G_{++}^0 & = -2s^2(t+u)C_{ttt}^{00}(s) - t_2 f_1(-t_2, u_2, h) \left[ C_{btt}^{h0}(t) + C_{tbb}^{h0}(t) \right] - t_1 f_1(-t_2, u_2, h) \\
& \times \left[ C_{btt}^{w0}(t) + C_{tbb}^{w0}(t) \right] + \left[ 2s\lambda m_b^2 + (N + sq)f_1(-t_2, u_2, h) \right] D_{bttb}^{h0w0}(t, u) \\
& + 2s\lambda m_b^2 D_{btbb}^{hw00}(s, t) + 2s \left[ 2swh - sm_b^2(t+u) + m_t^2 f_1(-t_2, u_2, h) \right] D_{tbtt}^{hw00}(s, t), \\
F_{+-}^0 & = 2s \{ 2wN + (t+u)[N - f_2(0, t, \lambda)] + qf_2(t, u, 2\lambda) + 2q^2(t_1 + u_1) \} C_{bbb}^{00}(s) \\
& - 2t_2[wh(t_1 + u_1) + qf_1(2u, 2h, t)] C_{btt}^{h0}(t) + 2t_2[w(hd - 2tt_2 - 2N) \\
& - qf_1(2u, 2h, t)] C_{tbb}^{h0}(t) - 2t_1[w(hd - 2ut_2) + qf_1(2t, 2h, t)] C_{btt}^{w0}(t) \\
& + 2t_1[w(hd - 2tt_2) - qf_1(2t, 2h, t)] C_{tbb}^{w0}(t) - 2\{\lambda(ud - 2wt_2) - (t_1 + u_1) \\
& \times [N(t+u) + q(d^2 + 2N)]\} C_{btt}^{hw}(s) - [ud - 2N - q(t_1 + u_1)] \left[ N(m_b^2 + m_t^2) + sq^2 \right] \\
& \times D_{bttb}^{h0w0}(t, u) - \left\{ 2N \left[ wN - m_b^2 f_3(t, u, 2w) \right] - Nq \left[ ud + 2u_1^2 + 2m_b^2(t_1 + u_1) \right] \right. \\
& \left. - q^2 [du_1(t_1 + u_1) - t_2 f_3(t, u, w)] - sq^3(t_1 + u_1) \right\} D_{tbtt}^{h0w0}(t, u) - 2\{stw(hd - 2tt_2) \\
& - 2Nm_b^2 f_2(0, t, \lambda) - q \left[ sN(t+u) - stf_2(t, 0, 2\lambda) + 2Nm_b^2(t_1 + u_1) \right] \\
& - sq^2 f_2(t, 0, \lambda) - sq^3(t_1 + u_1) \} D_{btbb}^{hw00}(s, t) + 2(t_1 + u_1) \left\{ stwh + 2uNm_t^2 \right. \\
& \left. + q \left[ st(t+2u) - N(s - 2m_t^2) \right] + sq^2(2t+u) + sq^3 \right\} D_{tbtt}^{hw00}(s, t), \\
G_{+-}^0 & = 2s \left[ (t+u)(d^2 + 2N) - 2\lambda q \right] C_{bbb}^{00}(s) + 2 \left[ t^2(t+u) - wh(3t-u) \right] \\
& \times \left\{ t_2 \left[ C_{btt}^{h0}(t) + C_{tbb}^{h0}(t) \right] + t_1 \left[ C_{btt}^{w0}(t) + C_{tbb}^{w0}(t) \right] \right\} - 2\lambda(d^2 + 2N) C_{btt}^{hw}(s) \\
& - \lambda \left[ N(m_b^2 + m_t^2) + sq^2 \right] D_{bttb}^{h0w0}(t, u) + \left[ 2N \left( 2wN - \lambda m_b^2 \right) - \lambda q(N + sq) \right] \\
& \times D_{tbtt}^{h0w0}(t, u) - 2f_4(m_b^2, m_t^2) D_{btbb}^{hw00}(s, t) - 2f_4(m_t^2, m_b^2) D_{tbtt}^{hw00}(s, t), \\
F_{++}^1 & = 2s^2 d \left[ m_b^2 C_{bbb}^{00}(s) - m_t^2 C_{ttt}^{00}(s) \right] - f_5(w, m_b^2, m_t^2) \left[ t_2 C_{btt}^{h0}(t) + t_1 C_{tbb}^{w0}(t) \right] \\
& + f_5(w, m_t^2, m_b^2) \left[ t_2 C_{tbb}^{h0}(t) + t_1 C_{btt}^{w0}(t) \right] + \left[ N + s(m_b^2 + m_t^2) \right] f_5(w, m_b^2, m_t^2) \\
& \times D_{bttb}^{h0w0}(t, u) + 2sf_6(m_b^2, m_t^2) D_{btbb}^{hw00}(s, t) - 2sf_6(m_t^2, m_b^2) D_{tbtt}^{hw00}(s, t), \\
F_{++}^2 & = -(N - sq) \left[ t_2 C_{btt}^{h0}(t) - t_1 C_{tbb}^{w0}(t) \right] + (N + sq) \left[ t_2 C_{tbb}^{h0}(t) - t_1 C_{tbb}^{w0}(t) \right] \\
& + \left\{ N \left[ N + 2s(m_b^2 + m_t^2) \right] + s^2 q^2 \right\} D_{bttb}^{h0w0}(t, u), \\
G_{++}^1 & = -sd(t_2 + u_2) C_{ttt}^{00}(s) + t_2 f_3(t, u, h) \left[ C_{btt}^{h0}(t) + C_{tbb}^{h0}(t) \right] - t_1 f_3(u, t, h) \\
& \times \left[ C_{btt}^{w0}(t) + C_{tbb}^{w0}(t) \right] + (N + sq)f_3(u, t, h) D_{bttb}^{h0w0}(t, u) + s(t_2 + u_2)[2N + d(t+q)] \\
& \times D_{tbtt}^{hw00}(s, t), \\
G_{++}^2 & = -sdC_{ttt}^{00}(s) + t_2 u_2 \left[ C_{btt}^{h0}(t) + C_{tbb}^{h0}(t) \right] + (st + N) C_{btt}^{w0}(t) - t_1 t_2 C_{tbb}^{w0}(t) \\
& + t_2(N + sq) D_{bttb}^{h0w0}(t, u) + s[2N + d(t+q)] D_{tbtt}^{hw00}(s, t), \\
F_{+-}^1 & = -4sdNB_0 \left( s, m_b^2, m_b^2 \right) - 2s \{ sd(t^2 + u^2) - N[4su + d(w-h) + \lambda]
\end{aligned}$$

$$\begin{aligned}
& -2sq[d(t+u) - 2N] + 2sdq^2\}C_{bbb}^{00}(s) + 2t_2 \left[ 2sN(t+u) + f_7(m_b^2, m_t^2) \right] C_{btt}^{h0}(t) \\
& - 2t_2 f_7(m_b^2, m_t^2) C_{tbb}^{h0}(t) - 2t_1 \left[ 2t_2 N(t_1 + u_1) + f_8(m_b^2, m_t^2) \right] C_{btt}^{w0}(t) \\
& + 2t_1 f_8(m_b^2, m_t^2) C_{tbb}^{w0}(t) - 2s\lambda[N - d(t+u-2q)]C_{btb}^{hw}(s) + f_5(h, m_t^2, m_b^2) \\
& \times [N(m_b^2 + m_t^2) + sq^2] D_{bttb}^{h0w0}(t, u) - \left\{ N^2 [f_3(t, u, w) + 2m_t^2(3s + w - h)] \right. \\
& - Nq[s\lambda - N(t_1 + u_1) - sd(t_1 + 2m_t^2)] + 2sNq^2(2s + u_2) + s^2dq^3 \left. \right\} D_{tbdt}^{h0w0}(t, u) \\
& - 2s(t-q) \left\{ Nf_3(u, t, h) - d[st(t-2m_t^2) - 2t_1 t_2 m_b^2 + sq^2] \right\} D_{btbb}^{hw00}(s, t) \\
& + 2s \left\{ stdN - (ud + \lambda)(st^2 + 2Nm_t^2) - q[sd(t(t+2u) - N) + 2st\lambda + 2dNm_t^2] \right. \\
& \left. - sq^2[d(2t+u) + \lambda] - sdq^3 \right\} D_{tbtt}^{hw00}(s, t), \\
F_{+-}^2 & = -4sNB_0(s, m_b^2, m_b^2) - 2s[s(t^2 + u^2) + N(t_2 - u_1) + 4Nm_b^2 - 2sq(t+u) + 2sq^2] \\
& \times C_{bbb}^{00}(s) - 2t_2[u_1N - f_9(t_2, m_b^2, m_t^2)]C_{btt}^{h0}(t) - 2t_2[u_1N + f_9(-t_2, m_t^2, m_b^2)] \\
& \times C_{tbb}^{h0}(t) + 2t_1 f_9(t_2, m_b^2, m_t^2) C_{btt}^{w0}(t) - 2t_1 f_9(-t_2, m_t^2, m_b^2) C_{tbb}^{w0}(t) + 2s[d^2(t+u) \\
& + N(t+3u) - 2q(d^2 + 2N)]C_{btb}^{hw}(s) - q[N^2 + 2sN(m_b^2 + m_t^2) + s^2q^2] D_{bttb}^{h0w0}(t, u) \\
& - [u_1N^2 + Nq(tu_1 + ut_2 + u_1^2 + 4sm_t^2) - 2sNq^2 + s^2q^3] D_{tbdt}^{h0w0}(t, u) \\
& - 2f_{10}(t_2, m_b^2, m_t^2) D_{btbb}^{hw00}(s, t) + 2f_{10}(s, m_t^2, m_b^2) D_{tbtt}^{hw00}(s, t), \\
G_{+-}^1 & = -2f_3(u, t, h) \left\{ 2sC_{bbb}^{00}(s) + t_1[C_{btt}^{w0}(t) + C_{tbb}^{w0}(t)] \right\} + 2t_2 f_3(t, u, h) \left[ C_{btt}^{h0}(t) + C_{tbb}^{h0}(t) \right] \\
& - [dN(t_1 + u_1) + s\lambda q] D_{tbdt}^{h0w0}(t, u) + 2s(t-q)f_3(u, t, h) D_{btbb}^{hw00}(s, t) \\
& - 2s(t+q)f_3(t, u, h) D_{tbtt}^{hw00}(s, t), \\
G_{+-}^2 & = 2t_2 \left\{ 2sC_{bbb}^{00}(s) + u_2[C_{btt}^{h0}(t) + C_{tbb}^{h0}(t)] + t_1[C_{btt}^{w0}(t) + C_{tbb}^{w0}(t)] \right\} \\
& + f_5(w, m_b^2, m_t^2) D_{tbdt}^{h0w0}(t, u) - 2st_2(t-q) D_{btbb}^{hw00}(s, t) - 2su_2(t+q) D_{tbtt}^{hw00}(s, t), \quad (6)
\end{aligned}$$

where we have used the auxiliary functions

$$\begin{aligned}
f_1(t, u, q) &= -w(t-u) + q(t_1 + u_1), \\
f_2(t, u, \lambda) &= \lambda - (3t+u)(t_1 + u_1), \\
f_3(t, u, h) &= 2N - (t-u)(u-h), \\
f_4(m_b^2, m_t^2) &= -st[N(t+u) - t\lambda] - s(m_b^2 - m_t^2)[N(t+u) - 2t\lambda] \\
& + \lambda \left[ 2Nm_b^2 + s(m_b^2 - m_t^2)^2 \right], \\
f_5(w, m_b^2, m_t^2) &= N(t+u-2w) - sd(m_b^2 - m_t^2), \\
f_6(m_b^2, m_t^2) &= -sm_b^2 [2N + d(t+m_b^2 - m_t^2)], \\
f_7(m_b^2, m_t^2) &= sd(t^2 + N) - (m_b^2 - m_t^2) [2std + N(3s + w - h)], \\
f_8(m_b^2, m_t^2) &= 2N^2 - d(st^2 - t_2 N) + (m_b^2 - m_t^2) [d(2st + N) - 2t_2 N],
\end{aligned}$$

$$\begin{aligned} f_9(t_2, m_b^2, m_t^2) &= st^2 - t_2 N - (m_b^2 - m_t^2)(2st + N), \\ f_{10}(t_2, m_b^2, m_t^2) &= -s(t + m_b^2 - m_t^2)[st(t + 2m_b^2 - 2m_t^2) + N(t_2 + 4m_b^2) + sq^2]. \end{aligned} \quad (7)$$

Here it is understood that all variables appearing on the right-hand sides are to be taken as independent. E.g.,  $N$  should be treated as independent of  $t$ ,  $u$ ,  $w$ , and  $h$ . Notice that the UV divergences of  $F_{+-}^1$  and  $F_{+-}^2$  cancel in the expression for  $\mathcal{M}_{+-\lambda_W}$  in Eq. (2). Finally, we remark that we recover the SM result for  $d\sigma(gg \rightarrow ZH)/dt$  due to one quark flavour [4] from Eq. (4) by substituting  $m_W = m_Z$ ,  $m_b = m_t$ , and  $\tan\beta = 1$  and adjusting the strengths of the axial-vector and Yukawa couplings. In particular, the contribution proportional to the weak vector coupling then vanishes as required by charge-conjugation invariance. This serves as a useful check for our analytical and numerical analyses.

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